Delayed-choice Measurement and Temporal Nonlocality

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We study for a composite quantum system with a quantum Turing architecture the temporal non-locality of quantum mechanics by using the temporal Bell inequality, which will be derived for a discretized network dynamics by identifying the subsystem indices with (discrete) parameter time. However, the direct "observation" of the quantum system will lead to no violation of the temporal Bell inequality and to consistent histories of any subsystem. Its violation can be demonstrated, though, for a delayed-choice measurement.

Key words: Bell Inequality; Delayed-choice Measurement; Quantum Erasure.

It has been realized from the beginning era of the quantum mechanics that the time can be described in quantum mechanics not by a self-adjoint operator but as a classical parameter in the state change defined by a unitary operator $\hat{U}(t)$. If the state change could be defined without consideration of abstract wavefunctions and their evolutions, the relationship between the state change and (classical) parameter time would become more illustrative. In this case we might ask whether the quantum-mechanical state change is local on the (classical) time axis. The state change could formally be described completely by a two-time correlation function, which refers to two points on the time axis. But this is not enough, because these correlations can be regarded, from their non-classical aspects, not as fingerprint of consistent histories [1] related to the temporal locality [2]. We will show this for a composite quantum system ("quantum network" [3]) with a quantum Turing architecture (see Fig. 1) [4, 5], utilizing structure of its internal correlations which emerge from a sequence of modular unitary transformations $\{\hat{U}_\mu\}_{\mu\in\mathrm{N}}$. In this way we can build a net of these correlations. By identifying the classical subsystem indices μ with the discrete "time", we will implement a dynamics for which the operator $\hat{U}_{2\mu}$ generates at the time 2μ the correlation between a certain reference subsystem S and the subsystem μ. Then the state of S at the time 2μ can be stored in the subsystem μ ("memory spins") by means of the respective correlation, e.g. $K_{zz}^{(S,\mu)} = \langle\hat{\sigma}_z^{(S)} \otimes \hat{\sigma}_z^{(\mu)}\rangle$, generated by a non-invasive measurement on (S, μ) (e.g. *CNOT*-operation). The delayed-measurement of the correlation $K_z^{(\mu_1,\mu_2)} = \langle\hat{\sigma}_z^{(\mu_1)} \otimes \hat{\sigma}_z^{(\mu_2)}\rangle$ then describes exactly the two-time correlation $C^{(S)}(t_1 = 2\mu_1;\ t_2 = 2\mu_2)$ of S by reinterpreting the correlation between the subsystems μ_1 and μ_2 as the two-time correlations for S [2]. This identification of the subsystem index with the time index en-

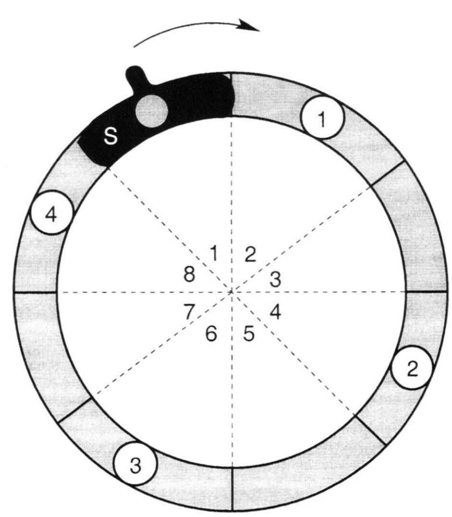

Fig. 1. Quantum network with a quantum Turing architecture: a reference subsystem S (Turing head) and memory spins $\mu = 1, 2, 3, 4$; step number (time) = 1, 2, ..., 8 (= position of S).

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ables us to construct the temporal Bell inequality, based on the standard Bell inequality with respect to subsystems μ, in order to test the non-locality of the quantum-mechanical time evolution. While the temporal Bell inequality cannot be violated by direct measurement, it will be shown that the inequality can be violated by post-selection (temporal which-path sorting) of the states of the subsystems μ [5]. One may thus conclude that the temporal locality and non-locality are complementary to each other just like in the spatial case.

The quantum network to be considered here will be composed of $M+1$ physically different $j=1/2$-spins $\nu = S, 1, 2, \ldots, M$ (here $M=4$). The respective spin states are $|p^{(\nu)}\rangle$, $p=-1, 1$. The corresponding product basis is $|p^{(S)}\, q^{(1)}\, r^{(2)}\, s^{(3)}\, t^{(4)}\rangle$. We can describe the total spin dynamics by using the $SU(2)$-algebra and cluster operators

$$\hat{K}_{jklmn} = \hat{\sigma}_j^{(S)} \otimes \hat{\sigma}_k^{(1)} \otimes \hat{\sigma}_l^{(2)} \otimes \hat{\sigma}_m^{(3)} \otimes \hat{\sigma}_n^{(4)}, \quad (1)$$

where $\hat{\sigma}_j$, $j = x, y, z$ are the Pauli matrices. Let the initial network state be $|\psi(0)\rangle = |-1^{(S)}\rangle\,|-1^{(1)}\rangle\,|-1^{(2)}\rangle\,|-1^{(3)}\rangle\,|-1^{(4)}\rangle$. In the first step we apply the local transformation $\hat{U}^{(S)}(\alpha) = \exp\left\{-\frac{i}{2}\alpha\,\hat{\sigma}_x^{(S)}\right\}$ with a phase α, and in the second step we excecute the CNOT-operation on $(S, 1)$ in order to generate the quantum correlation (= entanglement) between S and μ (only if $p(S) = -1$, the spin μ will flip). We thus get the strict anticorrelation between S and μ, $\hat{K}_{zz}^{(S,1)} = -1$. In the third step we again apply the local rotation $\hat{U}^{(S)}(\alpha)$ on S, which follows the CNOT on $(S, 2)$. In this way the spins $\mu = 1, 2, 3, 4$ will "keep", as memories, the states of S at the corresponding steps. Finally, we have at the 8th step

$$K_{zz}^{(1,2)} = K_{zz}^{(2,3)} = K_{zz}^{(3,4)} = \cos\alpha,\; K_{zz}^{(1,4)} = (\cos\alpha)^3. \quad (2)$$

Figure 2 shows the time evolution of spin S and the memory states at all steps. Via this built-in logic we interpret the correlations $K_{zz}^{(j,k)}$ as the two-time correlation functions $C^{(S)}(2j, 2k)$ between steps $2j$ and $2k$. The temporal Bell inequality reads

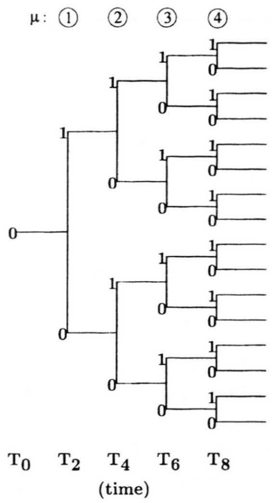
$$|K_{zz}^{(1,2)} + K_{zz}^{(2,3)} + K_{zz}^{(3,4)} - K_{zz}^{(1,4)}| \Rightarrow$$
$$|C^{(S)}(2,4) + C^{(S)}(4,6) + C^{(S)}(6,8) - C^{(S)}(2,8)| \le 2, \quad (3)$$

$$|\cos\alpha + \cos\alpha + \cos\alpha - \cos(\alpha+\alpha+\alpha)| \le 2. \quad (4)$$

However, this inequality cannot be violated [5]: each possible history of S can be considered an "element of reality" because subsequent measurements of the memories μ would project the subsystem S into an individual temporal trajectory of S describing a specific history of S. The perturbation by the non-invasive measurement at the steps 2 and 4 leads to a form of the correlation which differs from the quantum mechanical result $C^{(S)}(2,8) = \cos(3\alpha)$ to be expected for the isolated unitary dynamics of S. Therefore the temporal non-locality could be verified experimentally only by "incompatible" measurements of these two-time correlations. In Fig. 2 we see that a set of 4 measured values of spin components $(\sigma_z^{(1)}, \sigma_z^{(2)}, \sigma_z^{(3)}, \sigma_z^{(4)})$, $\sigma_z^{(\mu)} = \pm 1$ clearly allocates a specific history C_j, $j = 1, 2, \ldots, 2^4$. Similary we can also build another consistent history of S by considering $\sigma_x^{(\mu)} = \pm 1$ (not $\sigma_z^{(\mu)} = \pm 1$) with keeping one event of S at each step. In this case we get a different kind of history $C_j'(\sigma_x^{(1)}, \sigma_x^{(2)}, \sigma_x^{(3)}, \sigma_x^{(4)})$. Because of the incompatibility of $\hat{\sigma}_x^{(\mu)}$ and $\hat{\sigma}_z^{(\mu)}$ ($[\hat{\sigma}_x^{(\mu)}, \hat{\sigma}_z^{(\mu)}] \neq 0$) we cannot have the above two kinds of histories simultaneously. However, based on

$$|-1_x^{(\mu)}\rangle = (|-1_z^{(\mu)}\rangle - |1_z^{(\mu)}\rangle)/\sqrt{2},$$
$$|1_x^{(\mu)}\rangle = (|-1_z^{(\mu)}\rangle + |1_z^{(\mu)}\rangle)/\sqrt{2} \quad (5)$$

each history C_j' can be rewritten as a coherent superposition of all 2^4 histories C_j, $j = 1, 2, \ldots, 2^4$. Then it follows for each history C_j' that there is no projection of $\hat{\sigma}_z$ available on the σ_z-axis but only local rotations of spin S with respect to the σ_x-axis. Therefore all histories C_j' can be distinguished by the total rotating angle of



histories of S				memory states (T_8)			
T_8	T_6	T_4	T_2	④	③	②	①
1	1	1	1	0	0	0	0
0	1	1	1	1	0	0	0
1	0	1	1	0	1	0	0
0	0	1	1	1	1	0	0
1	1	0	1	0	0	1	0
0	1	0	1	1	0	1	0
1	0	0	1	0	1	1	0
0	0	0	1	1	1	1	0
1	1	1	0	0	0	0	1
0	1	1	0	1	0	0	1
1	0	1	0	0	1	0	1
0	0	1	0	1	1	0	1
1	1	0	0	0	0	1	1
0	1	0	0	1	0	1	1
1	0	0	0	0	1	1	1
0	0	0	0	1	1	1	1

Fig. 2. Alternative histories of S; $0 \equiv |-1\rangle$ and $1 \equiv |1\rangle$.

S: e.g. at the second step we have two histories for each case $C_j(C'_j)$, $j = 1, 2$ with the following network state

$$
\begin{aligned}
|\psi\rangle &= \cos(\alpha/2)\underbrace{|-1_z^{(S)}\rangle|1_z^{(\mu)}\rangle}_{C_1} \\
&\quad - i\sin(\alpha/2)\underbrace{|1_z^{(S)}\rangle|-1_z^{(\mu)}\rangle}_{C_2} \\
&= \frac{1}{\sqrt{2}}\Bigg(\exp\left\{-\frac{i}{2}\alpha\,\hat\sigma_x^{(S)}\right\}\underbrace{|-1_z^{(S)}\rangle\otimes|1_x^{(\mu)}\rangle}_{C'_1} \\
&\quad - \exp\left\{-\frac{i}{2}(-\alpha)\,\hat\sigma_x^{(S)}\right\}\underbrace{|-1_z^{(S)}\rangle\otimes|-1_x^{(\mu)}\rangle}_{C'_2}\Bigg). \qquad (6)
\end{aligned}
$$

Such a "weak" non-invasive measurement (leading to an ignorance of the temporal which-path information) enables us to reconstruct a unitary (coherent) dynamics of S (already having temporal which-path information by $\sigma_z^{(\mu)} = \pm 1$) as if the non-invasive measurement had not been applied to S. While both which-path markings by $\sigma_x^{(\mu)} = \pm 1$ and $\sigma_z^{(\mu)} = \pm 1$ generate in each case consistent internal histories, each history $C'_j(C_j)$ has no temporal which-path marking with respect to $\sigma_z^{(\mu)}$ $(\sigma_x^{(\mu)}) = \pm 1$. From this incompatibility of these markings a delayed-measurement of $\sigma_x^{(\mu)}$ $(\sigma_z^{(\mu)}) = \pm 1$ at the end step of the dynamics leads to the coherent superposition of $\sigma_z^{(\mu)}$ $(\sigma_x^{(\mu)}) = \pm 1$. Therefore the measurement of $\sigma_x^{(\mu)}$ erases the temporal which-path information of S marked by $\sigma_z^{(\mu)}$ by considering the ignorance of the "inter"-consistent histories between C_j and C'_j. Then we have no consistent histories any more. This back-action on the states in the past already indicates the temporally non-local aspect of quantum mechanics. Now we consider the correlation $K_{zz}^{(1,4)}$ in the Bell inequality. Before applying the delayed-measurement of the correlation functions we measure $\hat\sigma_x^{(2)}$, $\hat\sigma_x^{(3)}$ and select the corresponding network states, e.g. those with $\sigma_x^{(2)} = \sigma_x^{(3)} = +1$. After doing this we have the superposition of the four histories with

$$
\begin{aligned}
\sigma_z^{(2)} = \sigma_z^{(3)} = +1, \quad \sigma_z^{(2)} = -\sigma_z^{(3)} = +1, \\
\sigma_z^{(2)} = \sigma_z^{(3)} = -1, \quad \sigma_z^{(2)} = -\sigma_z^{(3)} = -1,
\end{aligned} \qquad (7)
$$

and the selected network state $|\psi_s\rangle$ reads

$$
\begin{aligned}
|\psi_s\rangle &= \cos(\alpha/2)\cdot\cos(3\alpha/2)\,|1_z^{(4)}\,1_x^{(3)}\,1_x^{(2)}\,1_z^{(1)}\rangle\,|-1^{(S)}\rangle \\
&\quad - i\cos(\alpha/2)\cdot\sin(3\alpha/2)\,|-1_z^{(4)}\,1_x^{(3)}\,1_x^{(2)}\,1_z^{(1)}\rangle\,|1^{(S)}\rangle \\
&\quad \sin(\alpha/2)\cdot\sin(3\alpha/2)\,|1_z^{(4)}\,1_x^{(3)}\,1_x^{(2)}\,(-1)_z^{(1)}\rangle\,|-1^{(S)}\rangle \qquad (8) \\
&\quad - i\sin(\alpha/2)\cdot\cos(3\alpha/2)\,|-1_z^{(4)}\,1_x^{(3)}\,1_x^{(2)}\,(-1)_z^{(1)}\rangle\,|-1^{(S)}\rangle.
\end{aligned}
$$

As a result 4 histories C_j with $\sigma_z^{(\mu)} = \pm 1$, $\mu = 2, 3$ have been erased (temporal quantum erasure) [6, 5]. Now we execute the delayed-measurement of $K_{zz}^{(1,4)}$ and finally get the correct quantum mechanical form of correlation function $K_{zz}^{(1,4)} = \cos(3\alpha)$ as if there had been no influence by the memories 2, 3 on the unitary dynamics of S. The same holds for other post-selected states, i.e. for other $\sigma_x^{(2)}$ and $\sigma_x^{(3)}$ results. In this way the violation of the temporal Bell inequality becomes measurable, e.g. with $\alpha = \pi/4$

$$
\begin{aligned}
&|\cos\alpha + \cos\alpha + \cos\alpha - \cos(3\alpha)| \\
&\quad = |3\cos(\pi/4) - \cos(3\pi/4)| = 2\sqrt{2} \nleq 2. \qquad (9)
\end{aligned}
$$

This violation shows explicitly that the quantum mechanical state change $\hat U(t)$ (before measurements) is, in general, temporally non-local and we therefore cannot expect consistent histories for a quantum-mechanical system under unitary evolution.

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